06-720 **Advanced Process Systems Engineering** **Spring 2011**

Homework 7 **Due: 3/30/11**

1. Given is a batch plant that manufactures 4 products A, B, C, D. It is desired to produce 2 batches of A, 2 batches of B, 5 batches of C and 4 batches of D. Assuming a zero-wait policy, determine a cyclic sequence with minimum cycle time. Processing times (hrs)

	Stage 1	Stage 2	Stage 3
A	5	4	2
B	7	5	4
C	5	6	2
D	8	8	2

Assume that clean-up times between different products can be neglected.

2. Determine the optimal sizes and number of parallel units for the following multiproduct batch problem operating with single product campaigns:.

Plant: 3 stages, 2 products
Demands: $Q_A = 200{,}000$, $Q_B = 100{,}000$ kg
Horizon: 6000 hrs
Cost units: $250V^{0.6}$ (V in L)
Lower bound volume: 250 L
Upper bound volume: 2500L
Maximum number of parallel units: 3

Size factors (L/kg)

	1	2	3
A	2	3	4
B	4	6	3

Processing times (hrs)

	1	2	3
A	8	20	8
B	16	4	4

3. Repeat problem 2, assuming that the demands for $Q_A = 80{,}000$ kg. $Q_B = 50{,}000$ kg., and that only one unit per stage is allowed. Determine the sizes required for single product campaigns, and for mixed product campaigns with ZW and UIS policy. In all cases clean-up times can be neglected.

4. Repeat problem 3 for the case of single product campaigns assuming that the equipment sizes are available as follows:

$$V = \{250, 750, 1000, 1500, 1750, 2500\} \text{ L}$$

How does your solution compare with the one in which the sizes obtained in problem 3 are rounded to the next highest value?

5. a) Draw the State-Task-Network for the case where three products A, B and C require mixing, reaction and separation operations in sequence (i.e. flowshop structure)

b) Use the processing times in Problem 2, formulate an STN problem that maximizes net sales (product sales – raw material costs).

6. Consider the NLP for the design of a multiproduct batch plant with one unit per stage, mixed product campaign and UIS:

$$
\begin{aligned}
Min \quad & \sum_{j=1}^{M} \alpha_j V_j^{\beta_j} \\
s.t. \quad & V_j \geq S_{ij} B_i, \qquad i = 1,\ldots N, j = 1,\ldots M \\
& \sum_{i=1}^{N} \frac{Q_i t_{ij}}{B_i} \leq H, j = 1,\ldots M \\
& V_j^L \leq V_j \leq V_j^U, j = 1,\ldots M, \qquad B_i \geq 0, i = 1,\ldots N
\end{aligned}
$$

where t_{ij} are the processing times for product i in stage j, S_{ij} are the size factors, and N and M are the number of products and stages, respectively.

a) Using an exponential transformation, show that this problem has a unique solution.

b) Extend this problem as an MINLP to also determine the optimal number of parallel units in each stage. Comment on the nature of the reformulation and solution.

06-720 **Advanced Process Systems Engineering**
Assignment 10 Solutions

1) 20 Point

No clean-up times

Slacks (SL_{1ik})

	A	B	C	D
A	0	0	0	0
B	0	0	0	0
C	1	0	1	0
D	3	1	3	0

Solution from LP:

	A	B	C	D
A	0	0	2	0
B	2	0	0	0
C	1	1	3	1
D	0	1	3	0

Sequence is

A → CCC → DDDD → B → A → CC → B

Cycle time is 85 hrs.

With clean-up times

Solution from LP ($NPRS_{ik}$)

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	2	1	1	1
D	4	2	4	0

Sequence is

$$\begin{tikzcd} \mathbf{AA} \arrow[r] & \mathbf{CCCCC} \arrow[r] & \mathbf{DDDD} \arrow[r] & \mathbf{BB} \arrow[lll, bend right] \end{tikzcd}$$

Cycle time is 90 hrs.

2) 25 Point

Cost = $ 106,755.84

Actual Volume:

Stage 1:	1200 lt.
Stage 2:	1800 lt.
Stage 3:	2400 lt.

No of units in parallel:

Stage 1:	2
Stage 2:	2
Stage 3:	1

Batch size of product:

A:	600 kg.
B:	300 kg.

Cycle times:

A:	10 hrs.
B:	8 hrs.

3) 25 Points

a)
One parallel equipment

Cycle time

$A = e^{2.996} = 20\ hrs.$
$B = e^{2.773} = 16\ hrs.$

Batch size:

$A = e^{6.279} = 533\ lt.$
$B = e^{5.586} = 266.66\ lt.$

Volume:

Stage 1:	$e^{6.972}$	= 1066.353 lt.
Stage 2:	$e^{7.378}$	= 1600.3 lt.
Stage 3:	$e^{7.665}$	= 2132.393 lt.

Cost = \$ 62,162.3161

b)
Unlimited intermediate storage multiproduct

Batch size:

$A = e^{5.768} = 319.89\ lt.$
$B = e^{5.298} = 199.93\ lt.$

Volume:

Stage 1:	$e^{6.685}$	= 800.31 lt.
Stage 2:	$e^{7.090}$	= 1199.90 lt.
Stage 3:	$e^{7.155}$	= 1280.492 lt.

Cost = \$ 49,686.9088

c)
Zero wait policy

Batch size:

A = 480 lt.
B = 240 lt.

No of batches:

$A = 166.667 \cong 167$
$B = 208.333 \cong 208$

Volume:

Stage 1:	960 lt.
Stage 2:	1440 lt.
Stage 3:	1920 lt.

Cost = $ 58,354.2771

Sequence:

A → B → A → B → A

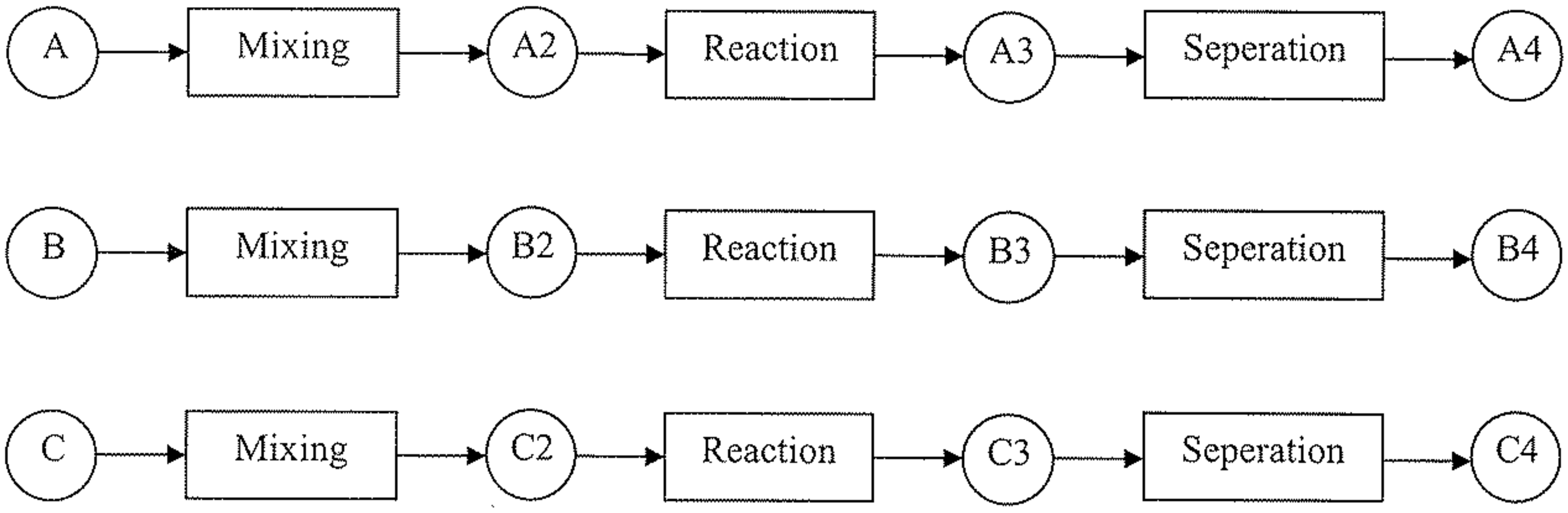
This sequence continues until A is exhausted.

4)   20 Points

Cost = $ 69,520

If the values obtained for the single product campaign is rounded off, then we have the same solution. The investment cost is higher because our system is overdesigned.

5) 10 Points


A
Mixing
A2
Reaction
A3
Seperation
A4
B
Mixing
B2
Reaction
B3
Seperation
B4
C
Mixing
C2
Reaction
C3
Seperation
C4

5 6) $A \rightarrow \boxed{1} \rightarrow A2 \rightarrow \boxed{2} \rightarrow A3 \rightarrow \boxed{3} \rightarrow A4$

$B \rightarrow \boxed{1} \rightarrow B2 \rightarrow \boxed{2} \rightarrow B3 \rightarrow \boxed{3} \rightarrow B4$

	1	2	3	(times x 4)
A	2	5	2	
B	4	1	1	

$i \in I_A^c = I_B^c = \{1\}$

$s \in \cancel{S} = \{A, A2, A3, A4, B, B2, B3, B4\}$

$i \in I_{Aj}^p = I_{Bj}^p = \{j\}$

$I_1 = \{1A, 1B\}$

$I_2 = \{2A, 2B\}$

$I_3 = \{3A, 3B\}$

$D_{st} = \{A4, B4\}$

$\Pi = \{A, B\}$

$p_{1j} = \{2, 4\}$

$p_{2j} = \{5, 1\}$

$p_{3j} = \{2, 1\}$

mass balance

A: $S_{At} = S_{At-1} - B_{1t} + \Pi_{At}$

B: $S_{Bt} = S_{Bt} - B_{1t} + \Pi_{Bt}$

A2: $S_{A2t} = S_{A2t-1} + B_{1t-2} - B_{2t}$

B2: $S_{B2t} = S_{B2t-1} + B_{1t-4} - B_{2t}$

A3: $S_{A3t} = S_{A3t-1} + B_{2t-5} - B_{3t}$

B3 $S_{B3t} = S_{B3t-1} + B_{2t-1} - B_{3t}$

A4 $D_{A4t} = D_{A4,t-1} + B_{3,t-2}$

B4 $D_{B4t} = D_{B4,t-1} + B_{3,t-1}$

Resource Balance

(treat A + B as resources, eqns (A) + (B)

Sequencing Constraints

Task 1

$$\sum_{t'=t-2+1}^{t} y_{1B,t'} \le 1 + M(1 - y_{1At})$$

$$\sum_{t'=t-4+1}^{t} y_{1A,t'} \le 1 + M(1 - y_{1Bt})$$

Task 2

$$\sum_{t'=t-5+1}^{t} y_{2B,t'} \le 1 + M(1 - y_{2At})$$

$$\sum_{t'=t-1+1}^{t} y_{2A,t'} \le 1 + M(1 - y_{1At})$$

Task 3

$$\sum_{t'=t-2+1}^{t} y_{3B,t'} \le 1 + M(1 - y_{3A,t})$$

$$\sum_{t'=t-1+1}^{t} y_{3A,t'} \le 1 + M(1 - y_{3B,t})$$

Can reformulate constraints as:

$$\sum_{i \in I_j} \sum_{\hat{t}=t-p_{ij}+1}^{t} y_{ij\hat{t}} \leq 1 \qquad \forall j, t$$

unit 1 A, B

$$\sum_{t'=t-1}^{t} (y_{1B,t'} + y_{1A,t'}) \leq 1$$

$$\sum_{t'=t-3}^{t} (y_{1B,t'} + y_{1A,t'}) \leq 1$$

unit 2 A, B

$$\sum_{t'=t-4}^{t} (y_{2B,t'} + y_{2A,t'}) \leq 1$$

$$\sum_{t'=t}^{t} (y_{2B,t'} + y_{2A,t'}) \leq 1$$

unit 3 A, B

$$\sum_{t'=t-1}^{t} (y_{3A,t'} + y_{3B,t'}) \leq 1$$

$$\sum_{t'=t}^{t} (y_{3A,t'} + y_{3B,t'}) \leq 1$$

<u>Objective function</u> (net sales)

$$\text{Max} \sum_{t=1}^{H} \left\{ (p_A D_{At} + p_B D_{Bt}) - (c_A \Pi_{At} + c_B \Pi_{Bt}) \right\}$$

6 NLP for VIS / MPC design

$$\text{Min} \sum_{j=1}^{M} \alpha_j V_j^{\beta_j}$$

$$\text{s.t.} \quad V_j \geq S_{ij} \cdot B_i$$

$$\sum_{i=1}^{N} \frac{Q_i t_{ij}}{B_i} \leq H$$

$$V_j^L \leq V_j \leq V_j^U$$

a) Redefine variables

$V_j = \exp v_j$ $\quad$ t_{ij}, Q_i constant

$B_i = \exp b_i$

$$\text{Min} \sum_{j=1}^{M} \alpha_j \exp(\beta_j v_j)$$

$$\text{s.t.} \quad v_j \geq \ln(S_{ij}) + b_i$$

$$\sum_{i=1}^{N} \exp(\ln(Q_i t_{ij}) - b_i) \leq H$$

$$v_j^L \leq v_j \leq v_j^U$$

Objective + constraint fcns. are strictly convex, so solution is global.

b) Parallel units formulation

$$\text{Min} \sum_{j=1} \alpha_j N_j V_j^{\beta_j}$$

$$V_j \geq S_{ij} B_i / N_i$$

$$\sum \frac{Q_i t_{ij}}{B_i} \leq H \qquad V_j^L \leq V_j \leq V_j^U$$

Again, transform variables:

$V_j = \exp v_j$ $\qquad t_{ij}, Q_i$ constant

$B_i = \exp b_i$

$N_j = \exp n_j$

$$\text{Min } \sum \alpha_j \exp(n_j + \beta_j V_j)$$

s.t.

$$v_j \geq \ln(S_{ij}) + b_i - n_j$$

$$\sum_i \exp(\ln(Q_i t_{ij}) - b_i) \leq H$$

$$v_j^L \leq v_j < v_j^U$$

Again, obj. + constraint fns are strictly convex, so relaxed solution is global.

The MINLP can be solved w/ NLP B+B and global relaxations can be found in tree. Also OA will always have feasible linearizations.