1. A two-boiler turbine-generator combination shown below

is used to produce a power output of 50 MW using a combination of fuel oil or blast furnace gas (BFG). However, to run these generators, BFG is limited to 12 units per hour and additional fuel oil needs to be purchased. To determine the amount of fuel needed, a curve fit has been done so that generator power \( p \) and fuel required \( f \) is given by
\[
f = a_0 + a_1 p + a_2 p^2.
\]
Coefficients for this correlation are given below.

<table>
<thead>
<tr>
<th>generator</th>
<th>fuel</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>oil</td>
<td>1.4609</td>
<td>0.15186</td>
<td>0.00145</td>
</tr>
<tr>
<td>1</td>
<td>gas</td>
<td>1.5742</td>
<td>0.16310</td>
<td>0.001358</td>
</tr>
<tr>
<td>2</td>
<td>oil</td>
<td>0.8008</td>
<td>0.20310</td>
<td>0.000916</td>
</tr>
<tr>
<td>2</td>
<td>gas</td>
<td>0.7266</td>
<td>0.22560</td>
<td>0.000778</td>
</tr>
</tbody>
</table>

Assume that the generators each produce power between 15 and 35.

(a) Based on the above information, formulate and solve an optimization problem to minimize the amount of fuel oil purchased using the GAMS solver.

(b) Does the above problem satisfy sufficient second order conditions? What is the reduced Hessian?

(c) If fuel oil is limited to 10 units per hour and BFG is purchased, What is the minimum BFG for 50 MW?

(d) For the above problem, what are the sensitivities of the optimum to a) increasing the power output, b) changing the limits of the generators?

2. Rederive the interior point method developed for \( \min f(x), \ s.t. \ c(x) = 0, x \geq 0 \) to the double-bounded NLP, \( \min f(x), \ s.t. \ c(x) = 0, x_l \leq x \leq x_u \).

(a) Extend the derivation of the primal-dual equations for \( \min f(x), \ s.t. \ c(x) = 0, x \geq 0 \) to the double-bounded NLP.

(b) Derive the resulting Newton step for these equations and the associate KKT matrix.
The document appears to be a technical specification or instruction sheet, possibly related to electrical or mechanical engineering. The text is not entirely legible due to the angle and quality of the image, but it seems to contain detailed lists, possibly of parameters or specifications, along with some notes and diagrams. The text includes terms and codes that suggest it is a detailed report or a set of instructions for a specific project or system. Without clearer visibility, it's challenging to provide a precise transcription or interpretation.
A small piece of text is written in a difficult-to-read handwriting, possibly containing a series of numbers and calculations or technical information. The handwriting is not legible enough to transcribe accurately. The content appears to be a continuation of a narrative or a list of specifications, but due to the quality of the handwriting, it is not possible to provide a clear and verbatim transcription.
2) Interior point method for

\[ \min \; f(x) \]

s.t. \( c(x) = 0 \)

\( x_L \leq x \leq x_u \)

KKT conditions

\[ \nabla f(x) + \nabla c(x) u + u_n - u_x = 0 \]

\( c(x) = 0 \)

\( 0 \leq x - x_L \perp u_x \geq 0 \)

\( 0 \leq x - x_u \perp u_n \geq 0 \)

a) primal-dual equations

\[ \nabla f(x) + \nabla c(x) u + u_n - u_x \]

\( c(x) = 0 \)

\( (x - x_L) u_x = \mu e \)

\( (x_u - x) u_n = \mu e \)

b) Linearizing primal-dual equations about \( x^k, u_x^k, u_n^k \)

\[
\begin{bmatrix}
    u_x^k & A_x & I & -I \\
    (A_x^k)^T & U_n^k & (x_u - x_x^k) & (x - x_x^k) \\
    U_n^k & (x_u - x_x^k) & u_n^k & (x - x_x^k) \\
    U_x^k & (x_u - x_x^k) & u_x^k & (x - x_x^k)
\end{bmatrix}
\begin{bmatrix}
    dx_x \\
    dx_n \\
    du_x \\
    du_n
\end{bmatrix}
= -
\begin{bmatrix}
    \nabla f(x^k) + \nabla c(x^k) u_x^k \\
    (x - x_x^k) u_n^k - \mu e \\
    (x_u - x_x^k) u_x^k - \mu e
\end{bmatrix}
\]

we can eliminate \( du_x \) due to create a symmetric KKT matrix
\[
\begin{pmatrix}
(u^2 + v^2 + w^2) A^2 & 0 \\
(A^2)^{-1} & 0
\end{pmatrix}
\quad \begin{pmatrix}
d_x \\
d_{uu}
\end{pmatrix} = \begin{pmatrix}
\nabla \phi_{mu}(x^k) + \nabla c(x^k) u^k \\
c(x^k)
\end{pmatrix}
\]

where
\[
Z = U_\mu (x^k - x^k)
\]
\[
Z_u = U_\mu (x^k - x^k) U_\mu^{-1}
\]
\[
\nabla \phi_{\mu} = \nabla \phi(x) - \mu (x^k - x^k) U_\mu(x^k - x^k)
\]

and
\[
d_{uu} = \mu (x^k - x^k) U_\mu^{-1} - (U_\mu^{-1} + (x^k - x^k) U_\mu U_\mu^{-1}) U_\mu U_\mu
\]

\[
d_{xx} = \mu (x^k - x^k) U_\mu^{-1} - (U_\mu^{-1} + (x^k - x^k) U_\mu U_\mu^{-1}) U_\mu U_\mu
\]