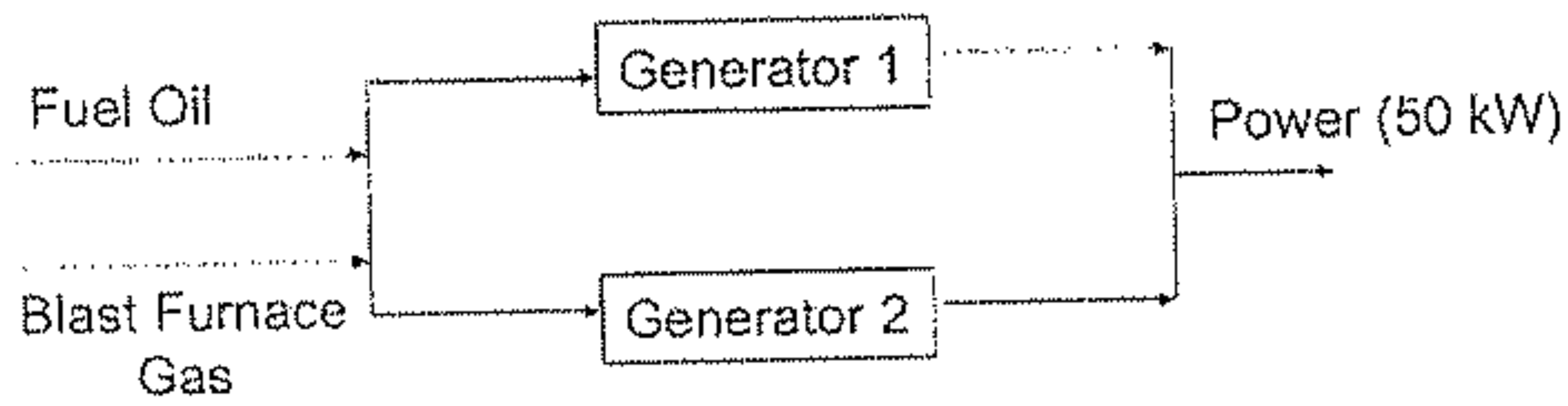
06-720 Advanced Process Systems Engineering Spring, 2011

Homework 4 Due: 2/9/11

1. A two-boiler turbine-generator combination shown below



is used to produce a power output of 50 MW using a combination of fuel oil or blast furnace gas (BFG). However, to run these generators, BFG is limited to 12 units per hour and additional fuel oil needs to be purchased. To determine the amount of fuel needed, a curve fit has been done so that generator power (x) and fuel required (f) is given by $f = a_0 + a_1 x + a_2 x^2$. Coefficients for this correlation are given below.

generator	fuel	a_0	a_1	a_2
1	oil	1.4609	0.15186	0.00145
1	gas	1.5742	0.16310	0.001358
2	oil	0.8008	0.20310	0.000916
2	gas	0.7266	0.22560	0.000778

Assume that the generators each produce power between 15 and 35.

(a) Based on the above information, formulate and solve an optimization problem to minimize the amount of fuel oil purchased using the GAMS solver.

(b) Does the above problem satisfy sufficient second order conditions? What is the reduced Hessian?

(c) If fuel oil is limited to 10 units per hour and BFG is purchased, What is the minimum BFG for 50 MW?

(d) For the above problem, what are the sensitivies of the optimum to a) increasing the power output, b) changing the limits of the generators?

2. Rederive the interior point method developed for $\min f(x), s.t.\ c(x) = 0, x \geq 0$ to the double-bounded NLP, $\min f(x), s.t.\ c(x) = 0, x_l \leq x \leq x_u$.

(a) Extend the derivation of the primal-dual equations for $\min f(x), s.t.\ c(x) = 0, x \geq 0$ to the double-bounded NLP.

(b) Derive the resulting Newton step for these equations and the associate KKT matrix.

1

Solution - Hwk 4

a)

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:45:12 Page 1
Power Generation via Fuel Oil
C o m p i l a t i o n
```

minimize oil purchased.

```

*OPTION SOLPRINT = OFF;

* Define index sets
SETS  G  Power Generators /gen1*gen2/
      F  Fuels             /oil, gas/
      K  Constants in Fuel Consumption Equations /0*2/;

* Define and Input the Problem Data
TABLE A(G,F,K) Coefficients in the fuel consumption equations
                  0          1          2

      gen1.oil    1.4609     .15186     .00145
      gen1.gas    1.5742     .16310     .001358
      gen2.oil    0.8008     .20310     .000916
      gen2.gas    0.7266     .22560     .000778;
PARAMETER PMAX(G) Maximum power outputs of generators /
      GEN1 35.0, GEN2 35.0/;
PARAMETER PMIN(G) Minimum power outputs of generators /
      GEN1 15.0, GEN2 15.0/;
SCALAR GASSUP Maximum supply of BFG in units per h /12.0/
       PREQ Total power output required in MW /50.0/;

* Define optimization variables
VARIABLES  P(G) Total power output of generators in MW
           X(G, F) Power outputs of generators from specific fuels
           Z(F) Total Amounts of fuel purchased
           OILPUR Total amount of fuel oil purchased;
POSITIVE VARIABLES P, X, Z;

* Define Objective Function and Constraints
EQUATIONS TPOWER Required power must be generated
          PWR(G) Power generated by individual generators
          OILUSE Amount of oil purchased to be minimized
          FUELUSE(F) Fuel usage must not exceed purchase;
TPOWER..  SUM(G, P(G)) =G= PREQ;
PWR(G).. P(G) =E= SUM(F, X(G,F));
FUELUSE(F).. Z(F) =G= SUM((K,G), a(G,F,K)*X(G,F)**(ORD(K)-1));
OILUSE.. OILPUR =E= Z("OIL");

* Impose Bounds and Initialize Optimization Variables
* Upper and lower bounds on P from the operating ranges
  P.UP(G) = PMAX(G);
  P.LO(G) = PMIN(G);
* Upper bound on BFG consumption from GASSUP
  Z.UP("gas") = GASSUP;
* Specify initial values for power outputs
  P.L(G) = .5*(PMAX(G)+PMIN(G));

* Define model and solve
  MODEL FUELOIL /all/;
  SOLVE FUELOIL USING NLP MINIMIZING OILPUR;

DISPLAY X.L, P.L, Z.L, OILPUR.L;
```

```
COMPILATION TIME     =        0.002 SECONDS      3 Mb  DII233-233 Dec 15, 2009
```

2

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:45:12 Page 5
Power Generation via Fuel Oil
Solution Report     SOLVE FUELOIL Using NLP From line 54

               S O L V E      S U M M A R Y

     MODEL   FUELOIL                OBJECTIVE  OILPUR
     TYPE    NLP                    DIRECTION  MINIMIZE
     SOLVER  CONOPT                 FROM LINE  54

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      2 Locally Optimal
**** OBJECTIVE VALUE               3.0403

 RESOURCE USAGE, LIMIT          0.033      1000.000
 ITERATION COUNT, LIMIT        10    2000000000
 EVALUATION ERRORS              0             0

    C O N O P T 3   version 3.14T
    Copyright (C)   ARKI Consulting and Development A/S
                    Bagsvaerdvej 246 A
                    DK-2880 Bagsvaerd, Denmark

 Using default options.

    The model has 9 variables and 6 constraints
    with 16 Jacobian elements, 4 of which are nonlinear.
    The Hessian of the Lagrangian has 4 elements on the diagonal,
    0 elements below the diagonal, and 4 nonlinear variables.

 ** Optimal solution. There are no superbasic variables.

 CONOPT time Total                            0.030 seconds
   of which: Function evaluations             0.001 =  3.7%
             1st Derivative evaluations       0.000 =  1.0%

 Workspace          =     0.03 Mbytes
    Estimate        =     0.03 Mbytes
    Max used        =     0.01 Mbytes

                       LOWER          LEVEL          UPPER         MARGINAL

---- EQU TPOWER        50.0000        50.0000        +INF            0.1661

  TPOWER  Required power must be generated

---- EQU PWR  Power generated by individual generators

          LOWER          LEVEL          UPPER         MARGINAL

gen1        .              .              .            -0.1661
gen2        .              .              .            -0.1688

                       LOWER          LEVEL          UPPER         MARGINAL

---- EQU OILUSE          .              .              .             1.0000

  OILUSE  Amount of oil purchased to be minimized

---- EQU FUELUSE  Fuel usage must not exceed purchase

          LOWER          LEVEL          UPPER         MARGINAL

oil         .              .            +INF            1.0000
gas         .              .            +INF            0.6782

---- VAR P  Total power output of generators in MW

          LOWER          LEVEL          UPPER         MARGINAL

gen1      15.0000        35.0000        35.0000          .
gen2      15.0000        15.0000        35.0000         0.0028
```

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:45:12 Page 6
Power Generation via Fuel Oil
Solution Report     SOLVE FUELOIL Using NLP From line 54

---- VAR X  Power outputs of generators from specific fuels

              LOWER          LEVEL          UPPER         MARGINAL

gen1.oil        .            4.8980         +INF             .
gen1.gas        .           30.1020         +INF             .
gen2.oil        .              .            +INF           0.0343
gen2.gas        .           15.0000         +INF             .

---- VAR Z  Total Amounts of fuel purchased

          LOWER          LEVEL          UPPER         MARGINAL

oil         .            3.0403         +INF             .
gas         .           12.0000        12.0000        -0.6782

                        LOWER          LEVEL          UPPER         MARGINAL

---- VAR OILPUR         -INF           3.0403         +INF             .

  OILPUR  Total amount of fuel oil purchased


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS
```

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:45:12 Page 7
Power Generation via Fuel Oil
E x e c u t i o n


----     56 VARIABLE X.L  Power outputs of generators from specific fuels

            oil         gas

gen1       4.898      30.102
gen2                  15.000


----     56 VARIABLE P.L  Total power output of generators in MW

gen1 35.000,    gen2 15.000


----     56 VARIABLE Z.L  Total Amounts of fuel purchased

oil  3.040,    gas 12.000


----     56 VARIABLE OILPUR.L             =        3.040  Total amount of fuel
oil purchased


EXECUTION TIME       =        0.001 SECONDS      3 Mb  DII233-233 Dec 15, 2009


USER: Ignacio E. Grossmann                               G100114/0001AP-DAR
      Carnegie Mellon University, Dept. of Chemical Engineering  DC8143
      License for teaching and research at degree granting institutions


**** FILE SUMMARY

Input      /Users/journal/Desktop/fueloil.gms
Output     /Users/journal/Desktop/fueloil.lst
```

b) from CONOPT solver, there are no superbasic variables. Hence SSOC is vacuously satisfied, and reduced Hessian is of zero dimension (no allowable directions / no null space).

c) minimize gas purchased

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:49:37 Page 1
Power Generation via Fuel Oil
C o m p i l a t i o n


*OPTION SOLPRINT = OFF;

* Define index sets
SETS  G  Power Generators /gen1*gen2/
      F  Fuels            /oil, gas/
      K  Constants in Fuel Consumption Equations /0*2/;

* Define and Input the Problem Data
TABLE A(G,F,K) Coefficients in the fuel consumption equations
                    0          1          2
      gen1.oil      1.4609     .15186     .00145
      gen1.gas      1.5742     .16310     .001358
      gen2.oil      0.8008     .20310     .000916
      gen2.gas      0.7266     .22560     .000778;
PARAMETER PMAX(G) Maximum power outputs of generators /
      GEN1 35.0, GEN2 35.0/;
PARAMETER PMIN(G) Minimum power outputs of generators /
      GEN1 15.0, GEN2 15.0/;
SCALAR OilUP Maximum supply of BFG in units per h /10.0/
       PREQ Total power output required in MW /50.0/;

* Define optimization variables
VARIABLES  P(G) Total power output of generators in MW
           X(G, F) Power outputs of generators from specific fuels
           Z(F) Total Amounts of fuel purchased
           BFGPUR Total amount of fuel oil purchased;
POSITIVE VARIABLES P, X, Z;

* Define Objective Function and Constraints
EQUATIONS TPOWER Required power must be generated
          PWR(G) Power generated by individual generators
          BFGUSE Amount of oil purchased to be minimized
          FUELUSE(F) Fuel usage must not exceed purchase;
TPOWER..  SUM(G, P(G)) =G= PREQ;
PWR(G).. P(G) =E= SUM(F, X(G,F));
FUELUSE(F).. Z(F) =G= SUM((K,G), a(G,F,K)*X(G,F)**(ORD(K)-1));
BFGUSE.. BFGPUR =E= Z("gas");

* Impose Bounds and Initialize Optimization Variables
* Upper and lower bounds on P from the operating ranges
  P.UP(G) = PMAX(G);
  P.LO(G) = PMIN(G);
* Upper bound on BFG consumption from GASSUP
  Z.UP("oil") = OILUP;
* Specify initial values for power outputs
  P.L(G) = .5*(PMAX(G)+PMIN(G));

* Define model and solve
  MODEL FUELOIL /all/;
  SOLVE FUELOIL USING NLP MINIMIZING BFGPUR;

DISPLAY X.L, P.L, Z.L, BFGPUR.L;


COMPILATION TIME     =        0.001 SECONDS      3 Mb  DII233-233 Dec 15, 2009
```

5

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:49:37 Page 5
Power Generation via Fuel Oil
Solution Report     SOLVE FUELOIL Using NLP From line 54

               S O L V E      S U M M A R Y

     MODEL   FUELOIL              OBJECTIVE  BFGPUR
     TYPE    NLP                  DIRECTION  MINIMIZE
     SOLVER  CONOPT               FROM LINE  54

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      2 Locally Optimal
**** OBJECTIVE VALUE                4.2541

 RESOURCE USAGE, LIMIT          0.019      1000.000
 ITERATION COUNT, LIMIT         7    2000000000
 EVALUATION ERRORS              0             0

    C O N O P T 3   version 3.14T
    Copyright (C)   ARKI Consulting and Development A/S
                    Bagsvaerdvej 246 A
                    DK-2880 Bagsvaerd, Denmark

 Using default options.

    The model has 9 variables and 6 constraints
    with 16 Jacobian elements, 4 of which are nonlinear.
    The Hessian of the Lagrangian has 4 elements on the diagonal,
    0 elements below the diagonal, and 4 nonlinear variables.

 ** Optimal solution. There are no superbasic variables.

 CONOPT time Total                            0.003 seconds
   of which: Function evaluations             0.000 =  8.8%
             1st Derivative evaluations       0.000 =  5.9%

 Workspace            =     0.03 Mbytes
    Estimate          =     0.03 Mbytes
    Max used          =     0.01 Mbytes

                          LOWER          LEVEL          UPPER         MARGINAL

---- EQU TPOWER          50.0000        50.0000         +INF            0.2008

  TPOWER  Required power must be generated

---- EQU PWR  Power generated by individual generators

            LOWER          LEVEL          UPPER         MARGINAL

gen1          .              .              .            -0.1929
gen2          .              .              .            -0.2008

                          LOWER          LEVEL          UPPER         MARGINAL

---- EQU BFGUSE             .              .              .             1.0000

  BFGUSE  Amount of oil purchased to be minimized

---- EQU FUELUSE  Fuel usage must not exceed purchase

            LOWER          LEVEL          UPPER         MARGINAL

oil           .              .            +INF            0.8708
gas           .              .            +INF            1.0000

---- VAR P  Total power output of generators in MW

            LOWER          LEVEL          UPPER         MARGINAL

gen1       15.0000        35.0000        35.0000         -0.0079
gen2       15.0000        15.0000        35.0000           .
```

(6)

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:49:37 Page 6
Power Generation via Fuel Oil
Solution Report     SOLVE FUELOIL Using NLP From line 54

---- VAR X  Power outputs of generators from specific fuels

                LOWER          LEVEL        UPPER       MARGINAL

gen1.oil          .           24.0265       +INF           .
gen1.gas          .           10.9735       +INF           .
gen2.oil          .           15.0000       +INF           .
gen2.gas          .              .          +INF         0.0248

---- VAR Z  Total Amounts of fuel purchased

            LOWER          LEVEL         UPPER       MARGINAL

oil           .           10.0000       10.0000      -0.8708
gas           .            4.2541       +INF           .

                          LOWER          LEVEL         UPPER       MARGINAL

---- VAR BFGPUR           -INF           4.2541        +INF           .

  BFGPUR  Total amount of fuel oil purchased


**** REPORT SUMMARY :          0     NONOPT
                               0 INFEASIBLE
                               0  UNBOUNDED
                               0     ERRORS
```

```
GAMS Rev 233  DII-DII 23.3.3 Mac Intel32/Darwin
02/07/11 15:49:37 Page 7
Power Generation via Fuel Oil
E x e c u t i o n


----      56 VARIABLE X.L  Power outputs of generators from specific fuels

            oil         gas

gen1      24.026      10.974
gen2      15.000


----      56 VARIABLE P.L  Total power output of generators in MW

gen1 35.000,    gen2 15.000


----      56 VARIABLE Z.L  Total Amounts of fuel purchased

oil 10.000,    gas  4.254


----      56 VARIABLE BFGPUR.L               =        4.254  Total amount of fuel
oil purchased


EXECUTION TIME       =        0.000 SECONDS      3 Mb  DII233-233 Dec 15, 2009


USER: Ignacio E. Grossmann                               G100114/0001AP-DAR
      Carnegie Mellon University, Dept. of Chemical Engineering  DC8143
      License for teaching and research at degree granting institutions


**** FILE SUMMARY

Input      /Users/journal/Desktop/fueloil.gms
Output     /Users/journal/Desktop/fueloil.lst
```

d) Optimal sensitivity

a) total power – marginal value ($u_{low} = 0.2008$)

for every increase in MW, 0.2008 units of additional gas are required.

b) increasing upper bnd on gen1 would [illegible]

Decreasing lower bnd of gen 2 would not affect results, [illegible]

2) Interior point method for

$$\begin{aligned} \text{Min} \quad & f(x) \\ \text{s.t.} \quad & c(x) = 0 \\ & x_l \le x \le x_u \end{aligned}$$

KKT conditions

$$\begin{aligned} \nabla f(x) + \nabla c(x) v + u_u - u_l &= 0 \\ c(x) &= 0 \\ 0 \le x - x_l \perp u_l &\ge 0 \\ 0 \le x_u - x \perp u_u &\ge 0 \end{aligned}$$

a) primal-dual equations

$$\begin{aligned} \nabla f(x) + \nabla c(x) v + u_u - u_l \\ c(x) = 0 \\ (X - X_l)\, u_l = \mu e \\ (X_u - X)\, u_u = \mu e \end{aligned}$$

b) Linearize primal-dual eqns about x^k, u_l^k, u_u^k, v^k

$$\begin{bmatrix} W^k & A^k & I & -I \\ (A^k)^T & & & \\ U_u^k & & (X_u - X^k) & \\ U_l^k & & & (X^k - X_l) \end{bmatrix} \begin{bmatrix} d_x \\ d_v \\ d_{u_u} \\ d_{u_l} \end{bmatrix} = - \begin{bmatrix} \nabla f(x^k) + \nabla c(x^k) v^k + u_u^k - u_l^k \\ c(x^k) \\ (X_u - X^k) u_u^k - \mu e \\ (X^k - X_l) u_l^k - \mu e \end{bmatrix}$$

we can eliminate d_{u_l} & d_{u_u} to create a symmetric KKT matrix

$$\begin{bmatrix} W^k + \Sigma_\ell + \Sigma_u & A^k \\ (A^k)^T & 0 \end{bmatrix} \begin{bmatrix} d_x \\ d_v \end{bmatrix} = \begin{bmatrix} \nabla \varphi_\mu(x^k) + \nabla c(x^k) v^k \\ c(x^k) \end{bmatrix}$$

where $\Sigma_\ell = U_\ell (X^k - X_\ell)^{-1}$

$\Sigma_u = U_u (X^k - X_u)^{-1}$

$\nabla \varphi_\mu = \nabla f(x) - \mu (X^k - X_\ell)^{-1} e - \mu (X^k - X_u)^{-1} e$

and $d_{u_u} = \mu (X_u - X^k)^{-1} e - (u_u^k + (X_u - X^k)^{-1} U_u^k d_x)$

$d_{u_\ell} = \mu (X^k - X_\ell)^{-1} e - (u_\ell^k + (X^k - X_\ell)^{-1} U_\ell^k d_x)$