1. Consider three common rules for the synthesis of distillation sequences.

\[ P_1 \land \neg P_2 \implies \neg P_3 \]
\[ \neg P_1 \land P_4 \implies P_5 \]
\[ P_2 \implies P_3 \]

where

- \( P_1 \) = lowest concentration component
- \( P_2 \) = most volatile component
- \( P_3 \) = remove component from top of column
- \( P_4 \) = easy component to separate
- \( P_5 \) = remove component first

(a) Write these logical expressions as English sentences.
(b) Rewrite rules in conjunctive normal form and write as constraints with binary variables.

2. Using GAMS solve the following MINLP problem step by step with

- a) Generalized Benders decomposition
- b) Outer-approximation method
- c) Extended cutting plane

Also verify your answer with GAMS/DICOPT.

\[
\text{min } f = y_1 + 1.5y_2 + 0.5y_3 + x_1^2 + x_2^2
\]

s.t. \((x_1 - 2)^2 - x_2 \leq 0\)
\[
x_1 - 2y_1 \geq 0
\]
\[
x_1 - x_2 - 4(1-y_2) \leq 0
\]
\[
x_1 - (1 - y_1) \geq 0
\]
\[
x_2 - y_2 \geq 0
\]
\[
x_1 + x_2 \geq 3y_3
\]
\[
y_1 + y_2 + y_3 \geq 1
\]
\[
0 \leq x_1 \leq 4, \quad 0 \leq x_2 \leq 4
\]
\[
y_1, y_2, y_3 = 0, 1
\]

Starting point \( y_1 = y_2 = y_3 = 1 \)

\[ x_1 = x_2 = x_3 = 0 \text{ for extended cutting plane.} \]
3. For the Generalized Disjunctive Program given below,
   a) Reformulate it as an MINLP using the convex hull formulation for the
disjunction
   b) Reformulate it as a big-M MINLP (M=50)
c) Solve both reformulations and compare their relaxations.

\[
\begin{align*}
\text{min } & \quad Z = c + (x_1 - 3)^2 + (x_2 - 2)^2 \\
\text{st } & \quad Y_i \\
& \quad x_1^2 + x_2^2 \leq 1 \vee (x_1 - 4)^2 + (x_2 - 1)^2 \leq 1 \vee (x_1 - 2)^2 + (x_2 - 4)^2 \leq 1 \\
& \quad c = 2 \, c = 1 \, c = 3 \\
& \quad 0 \leq x_1 \leq 8, \quad 0 \leq x_2 \leq 8, \quad Y_j = \text{true, false, } j = 1, 2, 3
\end{align*}
\]

4. It is proposed to manufacture a chemical C with a process I that uses
raw material B. B can either be purchased or manufactured with either
of two processes, II or III, which use chemical A as a raw material.
In order to decide the optimal selection of processes and levels of
production that maximize profit formulate the MINLP problem and solve
with the augmented penalty/outer-approximation/equality-relaxation
algorithm in DICOPT++.

Data:

Conversion:
Process I C = 0.9B
Process II B = ln(1 + A) Maximum capacity: 5 ton prod/hr
Process III B = 1.2 ln (1 + A) (A, B, C, in ton/hr)

Prices:
A $ 1,800/ton
B $ 7,000/ton
C $13,000/ton (maximum demand: 1 ton/hr)

Investment cost

<table>
<thead>
<tr>
<th>Process</th>
<th>Fixed (10^3$/hr)</th>
<th>Variable (10^3$/ton product)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3.5</td>
<td>2</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>1.5</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: Minimize negative of profit.