1. Use MATLAB to show that the solution profiles for the example:

\[
\begin{align*}
&\text{min} & -b(1) \\
&\text{s.t.} & \frac{da}{dt} = -a(t)(u(t) + 0.5u(t)^2) \\
& & \frac{db}{dt} = a(t)u(t) \\
& & a(0) = 1, \ b(0) = 0, \ u(t) \in [0, 5]
\end{align*}
\]

satisfy the optimality conditions.

![State and Control Profiles for Batch Reactor: A → B, A → C.](image)

Figure 1: State and Control Profiles for Batch Reactor: A → B, A → C.

2. For the singular control problem given by:

\[
\begin{align*}
&\text{min} & \Phi(z(t)), \\
&\text{s.t.} & \frac{dz(t)}{dt} = f_1(z) + f_2(z)u(t), z(0) = z_0, u(t) \in [u_L, u_U].
\end{align*}
\]

Show that \( q \geq 2 \) in \( \frac{d\varphi}{dq}(z, u) = 0 \).

3. For the problem below, derive the two point boundary value problem and show the relationship of \( u(t) \) to the state and adjoint variables.

\[
\begin{align*}
&\text{min} & z_1(1)^2 + z_2(1)^2 \\
&\text{s.t.} & \frac{dz_1(t)}{dt} = -2z_2, \ z_1(0) = 1 \\
& & \frac{dz_2(t)}{dt} = z_1u(t), \ z_2(0) = 1
\end{align*}
\]
4. Solve the Catalyst Example given below for the case where the first reaction is irreversible ($k_2 = 0$). Show that the solution is bang-bang.

\[
\begin{align*}
\text{min} & \quad a(t_f) + b(t_f) - a_0 \\
\text{s.t.} & \quad \frac{da(t)}{dt} = -u(k_1 a(t) - k_2 b(t)) \\
& \quad \frac{db(t)}{dt} = u(k_1 a(t) - k_2 b(t)) - (1 - u)k_3 b(t) \\
& \quad a(0) = a_0, \quad b(0) = 0, \quad u(t) \in [0, 1].
\end{align*}
\]