06-720 **Advanced Process Systems Engineering** **Spring, 2011**

. **Homework 5** **Due: 3/2/11**

1. For the system $xy = 1, x \in [1/2, 2], y \in [1/2, 2]$:
   (a) Plot the McCormick relaxation for this problem.
   (b) If the region is partitioned into four with $x \leq 1, x \geq 1$ and $y \leq 1, y \geq 1$, plot the resulting regions.
2. Solve the problem $max\ x + y, s.t.\ xy \leq 4,\ x \in [0, 6],\ y \in [0, 4]$.
3. Consider the following NLP:

$$\begin{aligned} min \quad & x_1 - x_2 - x_3 - x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4 \\ s.t. \quad & x_1 + 4x_2 \leq 8 \\ & 4x_1 + x_2 \leq 12 \\ & 3x_1 + 4x_2 \leq 12 \\ & 2x_3 + x_4 \leq 8 \\ & x_3 + 2x_4 \leq 8 \\ & x_3 + x_4 \leq 5 \\ & 0 \leq x_1, x_2, x_3, x_4 \leq 10 \end{aligned}$$

   (a) Apply McCormick convex envelopes and develop the LP lower bounding problems. Solve the problem to a global solution.
   (b) Verify the solution to this problem by solving it with BARON.
4. Consider the integer programming problem:

$$\begin{aligned} max \quad & 1.2y_1 + y_2 \\ s.t. \quad & y_1 + y_2 \leq 1 \\ & 1.2y_1 + 0.5y_2 \leq 1 \\ & y_1, y_2 = \{0, 1\} \end{aligned}$$

   (a) Determine from inspection the solution of the relaxed problem.
   (b) Enumerate the four 0-1 combinations in your plot to find the optimal solution.
   (c) Solve the relaxed LP problem by hand and derive Gomory cuts based on the LP relaxation. Verify that they cut-off the relaxed LP solution. *This part is optional and will not be graded. See the solution when posted.*

(d) Solve the above problem with the branch and bound method by enumerating the nodes in the tree and solving the LP subproblems with GAMS.

5. A company is considering to produce a chemical C which can be manufactured with either process II or process III, both of which use as raw material chemical B. B can be purchased from another company or else manufactured with process I which uses A as a raw material.

   Consider the two following cases:
   1. Maximum demand of C is 10 tons/hr with a selling price of $1800/ton.
   2. Maximum demand of C is 15 tons/hr; the selling price for the first 10 ton/hr is $1800/ton, and $1500/ton for the excess.

   Investment and Operating Costs:

	Fixed ($/hr)	Variable($/ton raw mat)
Process I	1000	250
Process II	1500	400
Process III	2000	550

   Prices:
   A: $500/ton
   B: $950/ton

   Conversions:

   Process I: 90% of A to B
   Process II: 82% of B to C
   Process III: 95% of B to C

   Maximum supply of A: 16 tons/hr

   Given the specifications above, formulate an MILP model and solve it with GAMS to decide:

   (a) Which process to build (II and III are exclusive)?
   (b) How to obtain chemical B?
   (c) How much should be produced of product C? The objective is to maximize profit.