1. For the system $xy = 1, x \in [1/2, 2], y \in [1/2, 2]$:
   (a) Plot the McCormick relaxation for this problem.
   (b) If the region is partitioned into four with $x \leq 1, x \geq 1$ and $y \leq 1, y \geq 1$, plot the resulting regions.

2. Solve the problem $\max x + y, s.t. xy \leq 4, x \in [0, 6], y \in [0, 4]$.

3. Consider the following NLP:
   \[
   \begin{align*}
   \min & \quad x_1 - x_2 - x_3 - x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4 \\
   \text{s.t.} & \quad x_1 + 4x_2 \leq 8 \\
   & \quad 4x_1 + x_2 \leq 12 \\
   & \quad 3x_1 + 4x_2 \leq 12 \\
   & \quad 2x_3 + x_4 \leq 8 \\
   & \quad x_3 + 2x_4 \leq 8 \\
   & \quad x_3 + x_4 \leq 5 \\
   & \quad 0 \leq x_1, x_2, x_3, x_4 \leq 10
   \end{align*}
   \]
   (a) Apply McCormick convex envelopes and develop the LP lower bounding problems. Solve the problem to a global solution.
   (b) Verify the solution to this problem by solving it with BARON.

4. Consider the integer programming problem:
   \[
   \begin{align*}
   \max & \quad 1.2y_1 + y_2 \\
   \text{s.t.} & \quad y_1 + y_2 \leq 1 \\
   & \quad 1.2y_1 + 0.5y_2 \leq 1 \\
   & \quad y_1, y_2 = \{0, 1\}
   \end{align*}
   \]
   (a) Determine from inspection the solution of the relaxed problem.
   (b) Enumerate the four 0-1 combinations in your plot to find the optimal solution.
   (c) Solve the relaxed LP problem by hand and derive Gomory cuts based on the LP relaxation. Verify that they cut-off the relaxed LP solution. This part is optional and will not be graded. See the solution when posted.
(d) Solve the above problem with the branch and bound method by enumerating the nodes in the tree and solving the LP subproblems with GAMS.

5. A company is considering to produce a chemical $C$ which can be manufactured with either process II or process III, both of which use as raw material chemical $B$. $B$ can be purchased from another company or else manufactured with process I which uses $A$ as a raw material. Consider the two following cases:

1. Maximum demand of $C$ is 10 tons/hr with a selling price of $1800/ton.
2. Maximum demand of $C$ is 15 tons/hr; the selling price for the first 10 ton/hr is $1800/ton, and $1500/ton for the excess.

### Investment and Operating Costs:

<table>
<thead>
<tr>
<th></th>
<th>Fixed ($/hr)</th>
<th>Variable($/ton raw mat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process I</td>
<td>1000</td>
<td>250</td>
</tr>
<tr>
<td>Process II</td>
<td>1500</td>
<td>400</td>
</tr>
<tr>
<td>Process III</td>
<td>2000</td>
<td>550</td>
</tr>
</tbody>
</table>

### Prices:
- A: $500/ton
- B: $950/ton

### Conversions:
- Process I: 90% of A to B
- Process II: 82% of B to C
- Process III: 95% of B to C

Maximum supply of A: 16 tons/hr

Given the specifications above, formulate an MILP model and solve it with GAMS to decide:

(a) Which process to build (II and III are exclusive)?
(b) How to obtain chemical B?
(c) How much should be produced of product C? The objective is to maximize profit.