

1. Construct the L/U algorithm using the Crout decomposition, using the same notation as in class.
2. For the following system of linear equations
 - a) Solve using Gauss elimination
 - b) Develop the LU factors and calculate the determinant
 - c) Resolve using L and U for the RHS provided

$$2x_1 + 4x_2 - x_4 = b_1$$

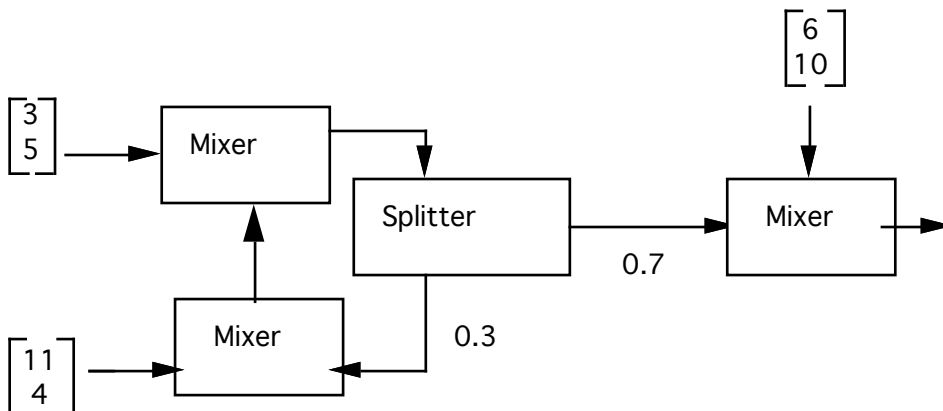
$$-x_3 + x_4 = b_2$$

$$x_1 + 6x_2 + 7x_3 = b_3$$

$$x_1 + x_4 = b_4$$

$$\underline{b}^T = (3, 2, 5, 4)$$

3. Retrieve the LU and QR decomposition programs from LINPACK in NETLIB and solve problem 2 again using both procedures. Alternately, MathCAD or MATLAB may be used.
4. Derive the linear balance equations for the problem



- a) Develop pivot sequences using the Markowitz criterion. Indicate the number of new non-zero elements created in developing L and U.
- b) For this case calculate L and U and solve the material balance equations.



1. Crout decomposition

- 1) let $r=1$
- 2) let $u_{rr} = 1$

3) Generate column r for L

$$l_{ir} = a_{ir} - \sum_{k=1}^{r-1} l_{ik} u_{kr} \quad i \geq r$$

4) Generate row r for U

$$u_{rj} = (a_{rj} - \sum_{k=1}^{r-1} l_{rk} u_{kj}) / l_{rr} \quad j \geq r$$

5) solve $Ly = b$
 $Ux = y$

$$y_i = (b_i - \sum_{j=1}^{i-1} l_{ij} y_j) / l_{ii} \quad i=1, n$$

$$x_i = (y_i - \sum_{j=i+1}^n u_{ij} x_j) \quad i=n \rightarrow 1$$

2 a) Gaussian elimination

$$2x_1 + 4x_2 - x_4 = 3$$

$$x_1 + 6x_2 + 7x_3 = 5$$

$$-x_3 + x_4 = 2$$

$$x_1 + x_4 = 4$$

$$\begin{bmatrix} 2 & 4 & 0 & -1 & 3 \\ 1 & 6 & 7 & 0 & 5 \\ 0 & 0 & -1 & 1 & 2 \\ 1 & 0 & 0 & 1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 0 & -1 & 3 \\ 0 & 4 & 7 & 1/2 & 7/2 \\ 0 & 0 & -1 & 1 & 2 \\ 0 & -2 & 0 & 3/2 & 5/2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 0 & -1 & 3 \\ 0 & 4 & 7 & 1/2 & 7/2 \\ 0 & 0 & -1 & 1 & 2 \\ 0 & 0 & 7/2 & 7/4 & 17/4 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 0 & -1 & 3 \\ 0 & 4 & 7 & 1/2 & 7/2 \\ 0 & 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 2 1/4 & 45/4 \end{bmatrix}$$

$$x_1 = 1.8571$$

$$x_2 = 0.3571$$

$$x_3 = 0.1428$$

$$x_4 = 2.1429$$

b) L/U decomposition

$$\begin{bmatrix} 2 & 4 & 0 & -1 \\ 1 & 6 & 7 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{matrix} L \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/2 & -1/2 & -7/2 & 1 \end{bmatrix} \end{matrix} \times$$

$$\det(A) = \det(L) \det(U) = (-42)(-1) = 42$$

U

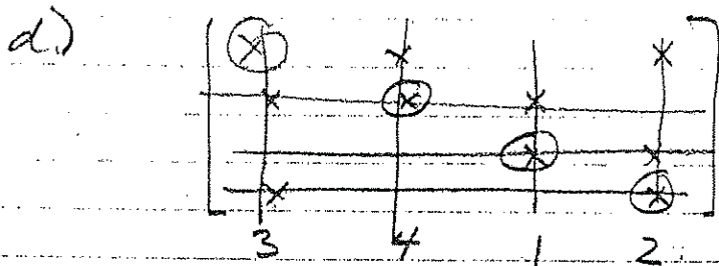
one row exchange = 42

c) alternate right hand side:

$$LU = \begin{bmatrix} 1 \\ 6 \\ 7 \\ 3 \end{bmatrix}$$

$$Ly = b \quad y = \begin{bmatrix} 129 \\ 5,5 \\ 7 \\ 29,75 \end{bmatrix}$$

$$Ux = y \quad x = \begin{bmatrix} -2,667 \\ 3,0 \\ -1,333 \\ 5,667 \end{bmatrix}$$



This sequence is not unique!

$$\begin{array}{c} 3 \\ 4 \\ 1 \\ 2 \end{array} \begin{bmatrix} 3 & 4 & 1 & 2 \\ -1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & -1 & 2 & 4 & 3 \\ 7 & 0 & 1 & 6 & 5 \end{bmatrix}$$

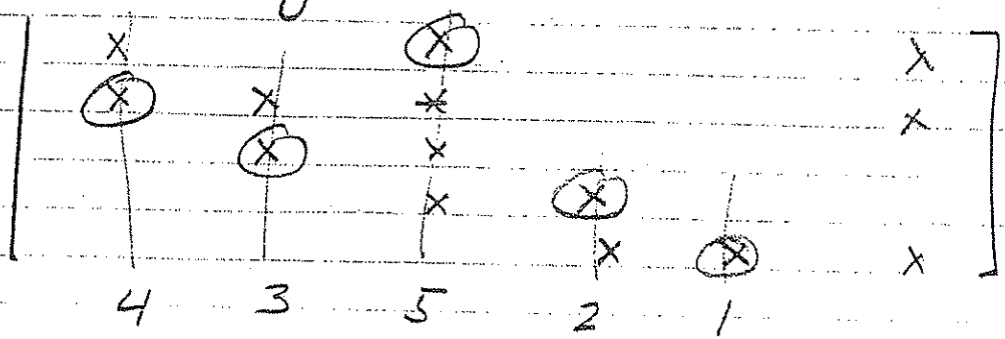
$$\rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & -1 & 2 & 4 & 3 \\ 0 & 7 & 1 & 6 & 19 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 3 & 4 & 7 \\ 0 & 0 & -6 & 6 & -9 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 3 & 4 & 7 \\ 0 & 0 & 0 & 14 & 5 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 1,8571 \\ x_2 &= 0,3571 \\ x_3 &= 0,14286 \\ x_4 &= 2,14286 \end{aligned}$$

3. Verify solutions for #2 with QR + LU.

Markowitz



Markowitz criterion $(\min(p_i - 1)(x_j - 1))$ can yield same sequence as min row / min col. for this problem. — nonunique

b) solution to mass balances problem.

$$\mu_{12} = \begin{bmatrix} 17 \\ 7.86 \end{bmatrix}$$

$$\mu_{13} = \begin{bmatrix} 6 \\ 3.86 \end{bmatrix}$$

$$\mu_{23} = \begin{bmatrix} 20 \\ 12.86 \end{bmatrix}$$

$$\mu_{34} = \begin{bmatrix} 14.0 \\ 9 \end{bmatrix}$$

$$\mu_{45} = \begin{bmatrix} 20 \\ 19 \end{bmatrix}$$