Computational Methods - Exam I

06-606 2/25/97

This 90 minute exam consists of five unequally weighted parts. Please budget your time carefully. You may consult your class notes, textbooks and homework sets to solve these problems.

1. *Algebraic Equation Solving*

   For the dogleg method, derive the Armijo line search with quadratic interpolation for this method. Show that the search direction is always a descent direction for $(1/2)f(x)^T f(x)$. (10 points)

2. *Broyden's Method*

   a) Assume that $(B^0)^{-1} = H^0$ is nonsingular. Derive a least distance updating formula for H^k that satisfies the secant relation. (10 points)

   b) Derive the analogous inverse update formula for B^k without using H^k in the final formula. (10 points)

3. *Runge-Kutta Methods*

   Consider the method given by the following Butcher block:

$$
\begin{array}{c|cc}
0 & 0 & 0 \\
2/3 & 1/3 & 1/3 \\
\hline
 & 1/4 & 3/4
\end{array}
$$

   a) Write the Runge-Kutta formula. Is it implicit or explicit? (5 points)

   b) What is the characteristic equation for stability for this method? (10 points)

   c) What are the stability properties of the method? Would you use this method for stiff systems? (10 points)

- over, please -

4. *Linear Multistep Methods*

a) The third order Adams-Bashforth method is given by:

$$y_{n+1} = y_n + (h/12)\,[23\, f_n - 16\, f_{n-1} + 5\, f_{n-2}];$$
$$\text{local error} = 9\, h^4\, f'''/24$$

while the corresponding Adams-Moulton method is given by:

$$y_{n+1} = y_n + (h/12)\,[5\, f_n + 8\, f_{n-1} - f_{n-2}];$$
$$\text{local error} = -h^4\, f'''/24$$

How would you estimate the local error at each step? Derive all necessary equations. (10 points)

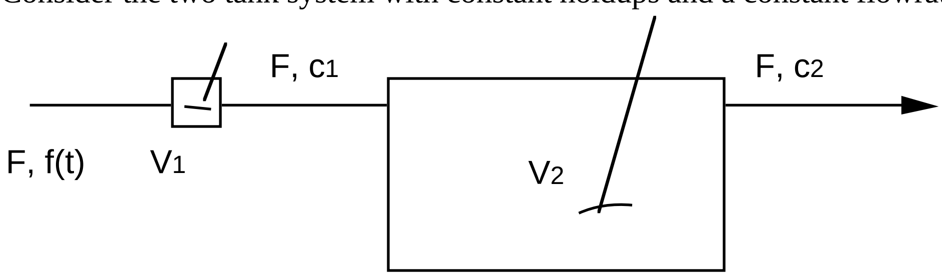
b) Consider the second order Gear corrector:

$$y_{n+1} = 4/3\, y_n - 1/3\, y_{n-1} + (2h/3)\, f_{n+1}$$

Determine the roots of the stability equation and calculate them for $h\lambda = 1$. (10 points)

5. *Stiff Equations*

Consider the two tank system with constant holdups and a constant flowrate.



where $\tau_1 = V_1/F$ and $\tau_2 = V_2/F$ and f(t) reflects the change in inlet concentration. The equations are given by:

$$\tau_1\, c_1' = -\, c_1 + f(t)$$
$$\tau_2\, c_2' = c_1 - c_2$$

a) If F = 10, $V_1 = 0.01$ and $V_2 = 1000$, calculate the stiffness ratio for this system. (5 points)

b) Is there any way that this problem can be reformulated so that it can be solved with explicit integrators? (10 points)

c) Suppose that 10% of the output of the second tank is recycled to the first tank. Modify the differential equations and recalculate the stiffness ratio, using the values in part a). Can the approach of part b) still be used? (10 points)