1. While searching for the minimum of

\[ f(x) = [(x_1^2 + (x_2 + 1)^2)/(x_1^2 + (x_2 - 1)^2)] \]

the algorithm terminates at the following points:

a) \( x^{(1)} = [0,0]^T \)
b) \( x^{(2)} = [0,1]^T \)
c) \( x^{(3)} = [0,-1]^T \)
d) \( x^{(4)} = [1,1]^T \)

Classify each point.

2. Consider the quadratic function with the parameter \( M \):

\[ f(x) = 3x_1 + x_2 + 2x_3 + 4x_1^2 + 3x_2^2 + 2x_3^2 + (M-2)x_1x_2 + 2x_2x_3 \]

For \( M = 0 \) find all stationary points. Are they optimal? Find the path of optimal solutions as \( M \) increases from zero.

3. In Powell damping, the BFGS update is modified if \( s^T y \) is not sufficiently positive by defining \( \bar{y} = \theta y + (1-\theta)B^k s \) and substituting for \( y \) in the BFGS formula.

a) Show that \( \theta \) can by found by solving the one-dimensional linear program:

\[ \max \theta \text{ s.t. } \theta s^T y + (1-\theta)s^T B^k s \geq 0.2 \ s^T B^k s, \theta \in [0,1] \]

b) If \( s^T y \geq 0.2 \ s^T B^k s \) show that Powell damping corresponds to a normal BFGS update.

c) If \( s^T y \rightarrow -\infty \), show that Powell damping corresponds to skipping the BFGS update.

4. Show that if \( B^k \) is positive definite \( \cos \theta^k > 1/\kappa(B^k) \) where \( \kappa(B^k) \) is the condition number of \( B^k \), based on the 2-norm.

5. Derive a stepsize rule for \( \alpha \) for the Armijo line search that minimizes the quadratic interpolant from the Armijo inequality.

6. Consider the convex problem:

\[ \min x_1 \text{ s.t. } x_2 \leq 0, x_2 - x_1^2 \geq 0 \]

Show that this problem does not satisfy LICQ and does not satisfy the KKT conditions at its optimum solution.
7. Consider the convex problem
\[
\min f(x) \quad s.t. \quad g(x) \leq 0
\]
and the equivalent problem
\[
\min f(x) \quad s.t. \quad g(x) + s = 0, s \geq 0.
\]

a) Show that the KKT conditions of the two problems are equivalent.
b) If the second problem has a local solution. Show that this is also a global solution.